

# ABOUT WHOLE NUMBERS

(E) a whole number (F) un nombre entier

## I. Generalities

- If one number can be divided exactly by another number, the second number is a **factor** of the first.

Example : The factors of 20 are 1 - 2 - 4 - 5 - 10 - 20

- **Multiples** are numbers that are in the multiplication tables.

Example : Five multiples of 8 are 8 - 24 - 56 - 80 - 1 600

- A **prime number** has only two factors, 1 and itself.

Example : The prime numbers up to 20 are 2 - 3 - 5 - 7 - 11 - 13 - 17 - 19.

Note that : 1 isn't a prime number ; 2 is the only even prime number.

History :

*Eratosthenes was a Greek mathematician (276 BC-194 BC).*



*He's famous for devising a method for finding prime numbers, called "the Sieve of Eratosthenes".*

*Euclid, another Greek mathematician (330 BC- 260 BC) proved that there are infinitely many prime numbers.*

Activity :

### The Sieve of Eratosthenes

- On a 100 square, cross out the number 1, since it isn't a prime number.
- Circle 2 and then cross out all other multiples of 2. The next number not crossed out is 3.
- Circle 3 and then cross out all other multiples of 3. The next number not crossed out is 5.
- Circle 5 and then cross out all other multiples of 5. The next number not crossed out is 7.
- Continue the process until no more numbers can be circled.

The circled numbers are all the prime numbers less than 100.

- **Indices**  $a^n$  : **a is the base** **n is the index (or power)**

Note that : index (in the singular) - indices (in the plural)

Example :  $7^5$  is read as "7 to the power of 5", it means  $7 \times 7 \times 7 \times 7 \times 7$ .

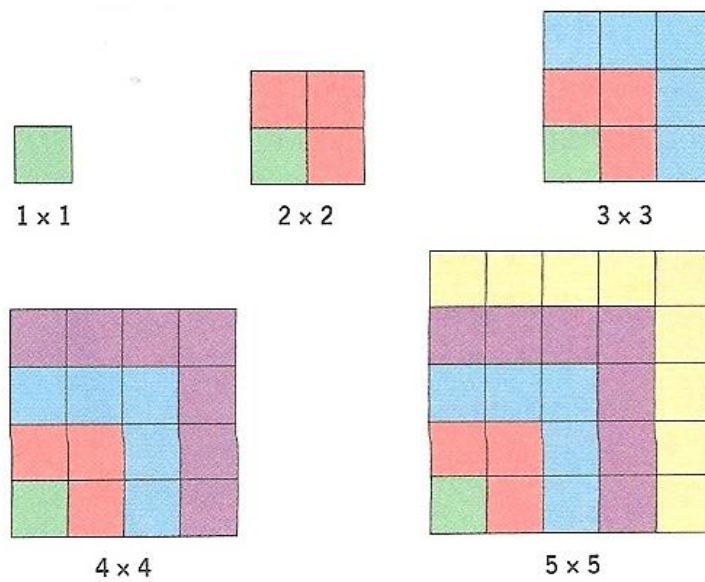
• **Square numbers** are whole numbers raised to the power of 2.

Examples :  $6^2 = 6 \times 6 = 36$  ("six squared")

(E) to square (F) prendre le carré de

1 - 4 - 9 - 16 - 25 - 36 - 49 - 64 - 81 - 100 are square numbers.

Square numbers can be illustrated by drawing squares:



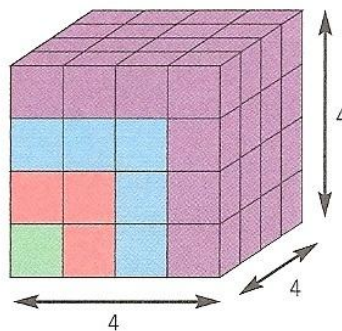
• **Cube numbers** are whole numbers raised to the power of 3.

Examples :  $4^3 = 4 \times 4 \times 4 = 64$  ("four cubed")

(E) to cube (F) prendre le cube de

1 - 8 - 27 - 64 - 125 - 216 are cube numbers.

You can draw a cube to illustrate cube numbers:



•  **$\sqrt{\quad}$  is the square root sign.** Taking the square root is the opposite of squaring.

Example :  $\sqrt{25} = 5$  since  $5^2 = 25$

(E) to take the square root (F) prendre la racine carrée de

- $\sqrt[3]{\quad}$  is the cube root sign. Taking the cube root is the opposite of cubing.

Example :  $\sqrt[3]{64} = 4$  since  $4^3 = 64$

(E) to take the cube root (F) prendre la racine cubique de

- The reciprocal of a number  $\frac{a}{b}$  is  $\frac{b}{a}$  (with a and b different from zero).

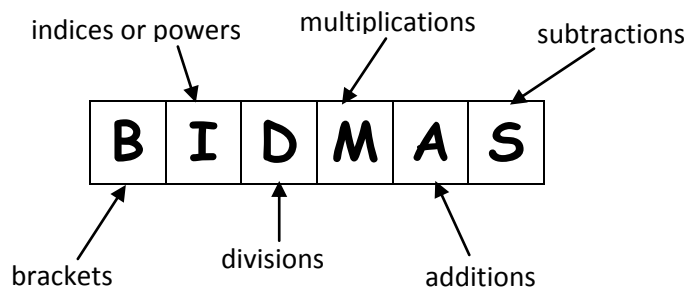
Example : The reciprocal of  $\frac{4}{9}$  is  $\frac{9}{4}$ .

(E) the reciprocal of a number (F) l'inverse d'un nombre

*Note that : Multiplying a number by its reciprocal gives 1 ; zero has no reciprocal.*

## II. BIDMAS

**BIDMAS** is a made-up word that helps you to remember the order in which calculations take place.



The brackets are worked out first, then division and multiplication are done before addition and subtraction.

Examples : Work out these expressions:

$$A = (3 + 5) \times 6$$

you carry out

$$A = 8 \times 6$$

the addition first

$$\underline{A = 48}$$

$$B = 3 + 5 \times 6$$

you carry out the

$$B = 3 + 30$$

multiplication first

$$\underline{B = 33}$$

$$C = (27 - 5^2) \times 2$$

you carry out the index first,

$$C = (27 - 25) \times 2$$

then the subtraction inside

$$C = 2 \times 2$$

the brackets

$$\underline{C = 4}$$

### III. PRIME FACTORS

**Prime factors** are factors that are prime numbers.

Some numbers can be written as the product of their prime factors.

Example : Decomposition of 30

$30 = 2 \times 15 = 2 \times 3 \times 5$  and 2 - 3 - 5 are prime numbers

### IV. HCF - LCM

#### 1. Highest Common Factor

The highest factor that two numbers have in common is called the **Highest Common Factor (HCF)**.

Example : Find the HCF of 60 and 84.

You write the numbers as a product of their prime factors.

$$60 = 2 \times 2 \times 3 \times 5$$

$$84 = 2 \times 2 \times 3 \times 7$$

You ring the common factors. These give the HCF as  $2 \times 2 \times 3 = \underline{12}$ .

#### 2. Lowest common multiple

The **Lowest Common Multiple (LCM)** is the lowest number that is a multiple of two or more numbers.

Example : Find the LCM of 14 and 20.

Write the numbers as a product of their prime factors, and ring the common factors.

$$14 = 2 \times 7$$

$$20 = 2 \times 2 \times 5$$

Line up the columns carefully.

The LCM of 14 and 20 is :  $2 \times 2 \times 7 \times 5 = \underline{140}$