About the Golden Ratio

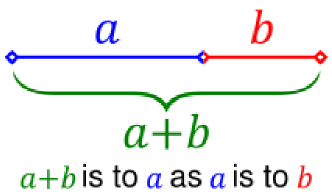
Definition

The **golden ratio** is a special number that appears many times in geometry, art, architecture and other areas.

Other names : *golden section, golden mean, golden number, divine proportion, divine section and golden proportion*.

*Symbol is the Greek letter "**phi**"* φ

The golden ratio (golden section) is defined for two lengths or quantities **a** and **b** when



$$\frac{a+b}{a} = \frac{a}{b} \equiv \varphi \,.$$

For example, if you divide a line into two parts so that:

the longer part divided by the smaller part

is also equal to

the whole length divided by the longer part

then you will have the golden ratio.

We have $\varphi = \dfrac{a+b}{a} = \dfrac{a}{a} + \dfrac{b}{a} = 1 + \dfrac{\mathbf{1}}{\varphi}$ so $\boxed{\varphi = 1 + \dfrac{\mathbf{1}}{\varphi}}$

The Golden Ratio is approximately 1.618

The digits just keep on going, with no pattern. The Golden Ratio is an irrational number.

That can be expanded into this fraction that goes on forever (called a *"continued fraction"*):

$$\varphi = 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{1+\cdots}}}$$

Calculating φ

You can calculate the Golden ratio yourself by starting with any number and following these steps:

● Divide 1 by your number (=1/number)

● Add 1

● That is your new number, start again at first step

With a calculator, just keep pressing "1/*x*", "+", "1", "=", around and around.

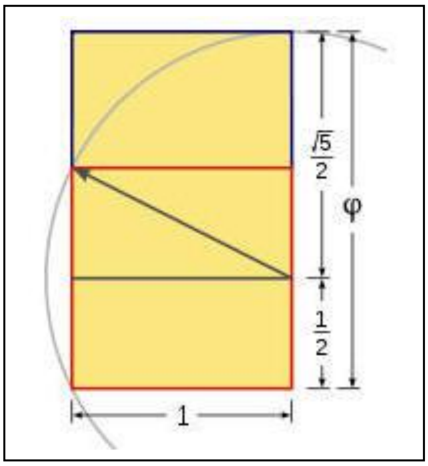
I started with 2 and got this:

Number	1/Number	Add 1
2	1/2=0.5	0.5+1=1.5
1.5	1/1.5 = 0.666...	0.666... + 1 = 1.666...
1.666...	1/1.666... = 0.6	0.6 + 1 = 1.6
1.6	1/1.6 = 0.625	0.625 + 1 = 1.625
1.625	1/1.625 = 0.6154...	0.6154... + 1 = 1.6154...
1.6154...		

It is getting closer and closer! But it takes a long time to get closer. However there are better ways and it can be calculated to thousands of decimal places quite quickly.

Golden Rectangle

A **golden rectangle** can be constructed with only straight edge and compass by this technique:



1) Construct a simple square

2) Draw a line from the midpoint of one side of the square

to an opposite corner

3) Use that line as the radius to draw an arc that defines

the height of the rectangle

4) Complete the golden rectangle

A **golden rectangle** is a rectangle whose side lengths are in the **golden ratio**, 1: φ (one-to-phi), that is approximately 1:1.618.

The Formula

Looking at the rectangle we just drew, you can see that there is a simple formula for it.

If one side is **1**, the other side will be:

$$\varphi = \frac{1}{2} + \frac{\sqrt{5}}{2} = \frac{1+\sqrt{5}}{2}$$

The square root of 5 is approximately 2.236068, so the Golden Ratio is approximately (1+2.236068)/2 = 3.236068/2 = 1.618034. This is an easy way to calculate it when you need it.

$$\boxed{\varphi = \frac{\mathbf{1}+\sqrt{\mathbf{5}}}{\mathbf{2}}}$$

The Fibonacci Sequence

Leonardo of Pisa, who was known as **Fibonacci**, introduced a sequence of numbers to western civilization in 1202.

This sequence is called the **Fibonacci sequence** or **Golden sequence.**

Starting with 0 and 1, each new number in the series is simply the sum of the two before it:

0 + 1 = 1   1 + 2 = 3   2 + 3 = 5   3 + 5 = 8   5 + 8 = 13   8 + 13 = 21 and so on.

The sequence looks like this: **0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89...**

There is a special relationship between the Golden Ratio and the Fibonacci sequence : if you take any two successive Fibonacci numbers, **their ratio is very close to the Golden Ratio**.

Let us try a few:

A	**B**	**B/A**
2	3	1.5
3	5	1.666666666...
5	8	1.6
8	13	1.625
...	...	...
144	233	1.618055556...
233	377	1.618025751...
...	...	...

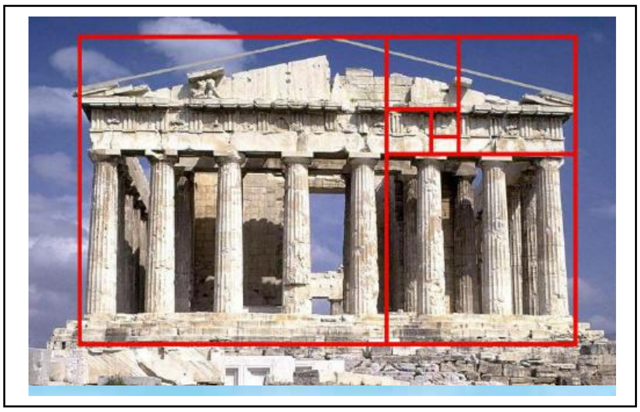
You don't even have to start with **2 and 3**. Here I chose **192 and 16** (and got the sequence *192, 16, 208, 224, 432, 656, 1088, 1744, 2832, 4576, 7408, 11984, 19392, 31376, ...*):

A	B	B / A
192	**16**	0.08333333...
16	208	13
208	224	1.07692308...
224	432	1.92857143...
...	...	...
7408	11984	1.61771058...
11984	19392	1.61815754...
...	...	...

Golden Ratio , Art & Nature

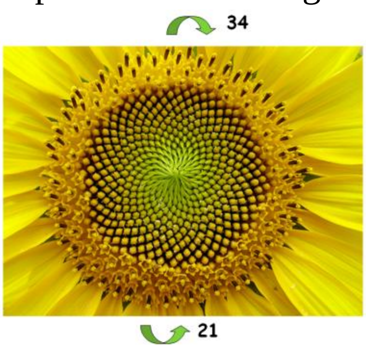
Some artists and architects believe the Golden Ratio makes the most pleasing and beautiful shape.

The Golden Ratio was used by the Egyptians to create their glorious pyramids, by the Greeks to design the famed Parthenon and by artists in the Renaissance as the measurement of all beauty.

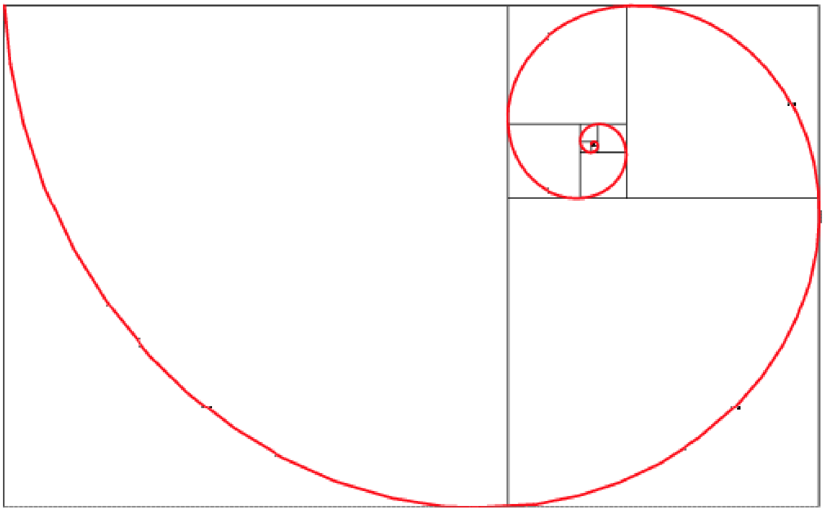


The Golden Ratio is seen in the proportions of the human body, animals, plants, DNA, the solar system and in the proportions of Art and Architecture.

Look at the array of seeds in the center of a sunflower and you'll notice what looks like spiral patterns curving left and right. Amazingly, if you count these spirals, your total will be a Fibonacci number. Divide the spirals into those pointed left and right and you'll get two consecutive Fibonacci numbers.



The Golden Spiral



The spiral is derived via the golden rectangle. When squared, it leaves a smaller rectangle behind, which has the same golden ratio as the previous rectangle. The squaring can continue indefinitely with the same result. No other rectangle has this trait.

The golden ratio is expressed in spiraling shells. In the above illustration, areas of the shell's growth are mapped out in squares. If the two smallest squares have a width and height of 1, then the box to their left has measurements of 2. The other boxes measure 3, 5, 8 and 13.

When you connect a curve through the corners of these concentric rectangles, you have formed the golden spiral. The Pythagoreans loved this shape for they found it everywhere in nature: the nautilus shell, ram's horns, milk in coffee, the face of a sunflower, your fingerprints, our DNA, and the shape of the Milky Way.

A slice through a Nautilus shell reveals golden spiral construction principle.

TIMELINE

● **Phidias** (490–430 BC) made the Parthenon statues that seem to embody the golden ratio.

● **Plato** (427–347 BC), in his *Timaeus*, describes five possible regular solids (the Platonic solids: the tetrahedron, cube, octahedron, dodecahedron, and icosahedron), some of which are related to the golden ratio.

● **Euclid** (c. 325–c. 265 BC), in his *Elements*, gave the first recorded definition of the golden ratio, which he called, as translated into English, "extreme and mean ratio" (Greek: ἄκρος καὶ μέσος λόγος).

● **Fibonacci** (1170–1250) mentioned the numerical series now named after him in his *Liber Abaci*. The ratio of sequential elements of the Fibonacci sequence approaches the golden ratio asymptotically.

● **Luca Pacioli** (1445–1517) defines the golden ratio as the "divine proportion" in his *Divina Proportione*.

● **Michael Maestlin** (1550–1631) publishes the first known approximation of the (inverse) golden ratio as a decimal fraction.

● **Johannes Kepler** (1571–1630) proves that the golden ratio is the limit of the ratio of consecutive Fibonacci numbers, and describes the golden ratio as a "precious jewel": "Geometry has two great treasures: one is the Theorem of Pythagoras, and the other the division of a line into extreme and mean ratio; the first we may compare to a measure of gold, the second we may name a precious jewel." These two treasures are combined in the Kepler triangle.

● **Charles Bonnet** (1720–1793) points out that in the spiral phyllotaxis of plants going clockwise and counter-clockwise were frequently two successive Fibonacci series.

● **Martin Ohm** (1792–1872) is believed to be the first to use the term *goldener Schnitt* (golden section) to describe this ratio, in 1835.

● **Edouard Lucas** (1842–1891) gives the numerical sequence now known as the Fibonacci sequence its present name.

● **Mark Barr** (20th century) suggests the Greek letter phi (**φ**), the initial letter of Greek sculptor Phidias's name, as a symbol for the golden ratio.

● **Roger Penrose** (b.1931) discovered a symmetrical pattern that uses the golden ratio in the field of aperiodic tilings, which led to new discoveries about quasicrystals.