Fractions
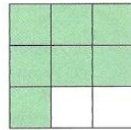
Fractions

A **fraction** is a whole unit divided into equal parts.

For example, $\frac{7}{9}$ means 7 parts out of 9:

Fractions are used every day. An example is shown below:

The top number is the **numerator**.

The bottom number is the **denominator**.

$$\frac{1}{2}$$

If the numerator is smaller than the denominator, it's called a **proper fraction**, for example, $\frac{7}{11}$.

If the numerator is bigger than the denominator, it's called an **improper fraction**, for example, $\frac{12}{7}$.

A fraction that has a whole number and a fraction is called a **mixed number**, for example, $3\frac{1}{2}$.

Equivalent Fractions

Equivalent fractions are fractions that have the same value.

Fractions can be changed into their equivalent by either multiplying or dividing the numerator and denominator by the same number.
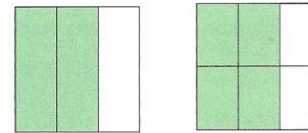
From the diagram, it can be seen that $\frac{2}{3} = \frac{4}{6}$.

Examples
Complete the equivalent fractions:

1 $\frac{3}{11} = \frac{?}{44}$

$$\frac{3}{11} \overset{\times 4}{\underset{\times 4}{=}} \frac{12}{44}$$

Multiply the numerator and denominator by 4.

2 $\frac{50}{60} = \frac{5}{?}$

$$\frac{50}{60} \overset{\div 10}{\underset{\div 10}{=}} \frac{5}{6}$$

Divide the numerator and denominator by 10.

Simplifying Fractions

Fractions can be **simplified** if the numerator and denominator have a common factor. This process is called **cancelling**.

$$\frac{20}{30} \overset{\div 10}{\underset{\div 10}{=}} \frac{2}{3}$$

10 is the **highest common factor** of 20 and 30, so dividing the numerator and denominator by 10 gives $\frac{2}{3}$.

Adding and subtracting fractions

Adding fractions

- You can only **add fractions** and **subtract** fractions if they have the same denominator.

Example: Add $\frac{2}{3} + \frac{3}{4}$

First, find the **lowest common multiple** (LCM) of the two denominators, 3 and 4.
The LCM is the smallest common number in the 3 and 4 times table.
This is 12.

Now make both fractions into twelfths.

$\frac{2}{3} = \frac{8}{12}$ (Multiply top and bottom by 4)

$\frac{3}{4} = \frac{9}{12}$ (Multiply top and bottom by 3)

Then just add the numerators and leave the denominator unchanged.

$\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$

A fraction like $\frac{17}{12}$ is called **top heavy**. It can be made into a **mixed number** $1\frac{5}{12}$.

Subtracting fractions

Example: Work out $\frac{5}{9} - \frac{1}{6}$

First, find the **lowest common multiple** (LCM) of the two denominators, 9 and 6. The LCM is the smallest common number in the 9 and 6 times table. This is 18.

Now make both fractions into eighteenths.

$\frac{5}{9} = \frac{10}{18}$ (Multiply top and bottom by 2)

$\frac{1}{6} = \frac{3}{18}$ (Multiply top and bottom by 3)

Then just subtract the numerators and leave the denominator unchanged.

$\frac{5}{9} - \frac{1}{6} = \frac{10}{18} - \frac{3}{18} = \frac{7}{18}$

> **Top Tip!**
>
> Subtracting a fraction from 1 is a common question, e.g.
>
> $1 - \frac{11}{14} = \frac{3}{14}$
>
> $1 - \frac{7}{9} = \frac{2}{9}$
>
> $1 - \frac{8}{11} = \frac{3}{11}$
>
> You should be able to see the relationship between the numerators and denominators.

Spot Check

1 Work out **a** $\frac{3}{5} + \frac{1}{4}$ **b** $\frac{3}{4} - \frac{1}{6}$

2 Work out **a** $1\frac{1}{4} + 2\frac{2}{3}$ **b** $3\frac{3}{5} - 1\frac{3}{8}$

Mixed numbers

- **Top-heavy** fractions can be converted into **mixed numbers**, which are a combination of a **whole number** and a **fraction**.

Example: Convert **a** $\frac{13}{5}$ to a mixed number **b** $3\frac{2}{9}$ to a top-heavy fraction.

   a Divide 13 by 5
     $13 \div 5 = 2$ rem 3
     So, $\frac{13}{5} = 2\frac{3}{5}$

   b Multiply 3 by 9 and add 2
     $3 \times 9 + 2 = 29$
     So, $3\frac{2}{9} = \frac{29}{9}$

- When adding and subtracting mixed numbers, they can be converted to top-heavy fractions or split into whole numbers and fractions, which makes the calculations easier.

Example: Work out **a** $3\frac{2}{5} + 1\frac{3}{4}$ **b** $4\frac{1}{6} - 2\frac{2}{9}$

   a $3\frac{2}{5} + 1\frac{3}{4} = 3 + 1 + \frac{2}{5} + \frac{3}{4} = 4 + \frac{8}{20} + \frac{15}{20} = 4 + \frac{23}{20} = 4 + 1\frac{3}{20} = 5\frac{3}{20}$

   b $4\frac{1}{6} - 2\frac{2}{9} = 4 - 2 + \frac{1}{6} - \frac{2}{9} = 2 + \frac{3}{18} - \frac{4}{18} = 2 + -\frac{1}{18} = 1\frac{17}{18}$

Sample mental test questions

Look at the fraction $\frac{4}{12}$. Write it in its simplest form.
The common factor is 4, so divide top and bottom by 4 to get the answer of $\frac{1}{3}$.

Add a half and three quarters.
The fractions will be really easy, so you should know that $\frac{1}{2} + \frac{3}{4} = 1\frac{1}{4}$.

Sample worked test question

A vegetable plot is planted with beans, peas, cabbages and carrots.

The peas take up $\frac{1}{4}$ of the plot.

The beans take up $\frac{3}{8}$ of the plot.

The cabbages take up $\frac{1}{6}$ of the plot.

How much of the plot is planted with carrots?

Answer
The total planted with peas, beans and cabbages is:

$\frac{1}{4} + \frac{3}{8} + \frac{1}{6}$

The common denominator is 24.

Making all the fractions into fractions with a denominator of 24 gives:

$\frac{6}{24} + \frac{9}{24} + \frac{4}{24} = \frac{19}{24}$ So $1 - \frac{19}{24} = \frac{5}{24}$ is planted with carrots.

Did You Know?

The population of the world increases by $2\frac{2}{3}$ people every second.

Multiplying and dividing fractions

Multiplying fractions

levels 5-6

- Adding and subtracting fractions requires denominators to be the same but multiplying and dividing fractions is a little more straightforward.

Example: Multiply **a** $\frac{2}{3} \times \frac{1}{4}$ **b** $1\frac{1}{4} \times 1\frac{7}{15}$

When multiplying fractions, the new numerator is the product of the numerators and the new denominator is the product of the denominators.

a Multiplying the numerators gives $2 \times 1 = 2$.
Multiplying the denominators gives $3 \times 4 = 12$.
So $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$.
This fraction $\frac{2}{12}$ will cancel to $\frac{1}{6}$.
To avoid problems with cancelling, cancel any fractions before multiplying:

$$\frac{\overset{1}{\cancel{2}}}{3} \times \frac{1}{\underset{2}{\cancel{4}}} = \frac{1}{6}$$

In this case, 2 on the top cancels with 4 on the bottom by a common factor of 2.

b Write both mixed numbers as **top-heavy** fractions, **cancel common factors** top and bottom and multiply the numerators and denominators. Finally, change the top-heavy answer back into a mixed number.

$$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{11}{\cancel{22}}}{\underset{3}{\cancel{15}}} = \frac{11}{6} = 1\frac{5}{6}$$

Top Tip!
Always write mixed numbers as top-heavy fractions when multiplying or dividing.
$3\frac{3}{4} = \frac{15}{4}$ because there are $3 \times 4 = 12$ quarters in 3 plus the extra 3 quarters.

Dividing fractions

level 6

- When dividing fractions, turn the **second** fraction **upside down** and **multiply**.

Example: Divide **a** $\frac{5}{6} \div \frac{1}{3}$ **b** $2\frac{1}{4} \div 1\frac{7}{8}$

a Write $\frac{5}{6} \div \frac{1}{3}$ as $\frac{5}{6} \times \frac{3}{1}$
Once again, cancel if you can. In this case, 3 and 6 cancel by a common factor of 3.

$$\frac{5}{\underset{2}{\cancel{6}}} \times \frac{\overset{1}{\cancel{3}}}{1} = \frac{5}{2} = 2\frac{1}{2}$$

Remember to change the top-heavy answer back into a mixed number.

b Write both mixed numbers as **top-heavy** fractions before turning the **second upside down** and **multiplying**. Then cancel common factors top and bottom and multiply the numerators and denominators. Finally, change the top-heavy answer back into a mixed number.

$$\frac{9}{4} \div \frac{15}{8} = \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{5}{\cancel{15}}} = \frac{6}{5} = 1\frac{1}{5}$$

Top Tip!
Always cancel before multiplying the numerators and denominators. It makes the calculations much easier.

Sample mental test questions

What is half of one third?
The fractions will be really easy, so you should know that $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

How many fifths are there in 2?
There are 5 fifths in 1, so there are 10 fifths in 2.

Sample worked test question

level 6

Work out $3\frac{3}{4} \div \frac{5}{8}$

Answers
Write the calculation as $\frac{15}{4} \div \frac{5}{8}$

Now turn the second fraction upside down and multiply
Cancel where possible.

$$\frac{\overset{3}{\cancel{15}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{5}}} = \frac{6}{1}$$

The calculation is $3\frac{3}{4} \div \frac{5}{8} = 6$

Top Tip!
Any fraction with a denominator of 1 is a whole number.

Did You Know?
One mile in every five of the US motorway network has to be straight so that it can be used as an airstrip in emergencies.

Spot Check

1 Work out **a** $\frac{5}{8} \times \frac{4}{15}$ **b** $2\frac{2}{3} \times 1\frac{1}{8}$

2 Work out **a** $\frac{2}{9} \div \frac{8}{15}$ **b** $2\frac{4}{5} \div 2\frac{1}{10}$